Government College of Engineering, Aurangabad

(An autonomous Institute of Government of Maharashtra)

T.E. (Mech) Examination

End semester Examination

ME 341: Applied Mathematics

12 NOV 2016

Time: Three hours *Max. Marks: 60*

"Verify the course code and check whether you have got the correct question paper".

N.B:-

1. All questions are compulsory
2. Figures to the right indicate full marks
3. Assume suitable data if necessary and state it clearly
4. Use of non programmable calculator is allowed

Q 1: A) Attempt the following (2)

i. State Cauchy Integral theorem.
ii. Define harmonic function.

B) Attempt any two (6)

i. Show that $u = logr$ is harmonic, find its harmonic conjugate and corresponding analytic function.
ii. Evaluate $\int_c \bar{z}\, dz$, where c is the circle $|z-2| = 3$.
iii. Evaluate $\int_c \frac{e^{2z}}{(z+1)^4}\, dz$, where $|z| = 2$ by Cauchy integral formula.

C) Attempt any one (4)

i. Determine the analytic function $f(z) = u + iv$ if $u - v = \frac{cosx+sinx-e^{-y}}{2[cosx-coshy]}$ & $f\left(\frac{\pi}{2}\right) = 0$.
ii. Prove that the analytic function with constant modulus is constant.

Q 2: A) Attempt the following (2)

i. State the formula for false position method.
ii. State the formula for Newton – Raphson method.

B) Attempt any two of the following (6)

i. Solve the equations by Jordan method $2x - y + z = 5, x + 3y - 2z = 7, x + 2y + 3z = 10$.
ii. Solve by Gauss - elimination method $x + 4y - z = -5, x + y - 6z = -12, 3x - y - z = 4$.
iii. Use Newton – Raphson method to find the real root of the equation $x^3 + 2x^2 + 10x - 20 = 0$.

C) Attempt any one (4)

i. Solve by Gauss - seidal method $2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$.
ii. Find the real root of the equation by regula false method $x^2 - log_e x = 12$.

Q 3: A) Attempt the following (2)

i. State simpson's $\left(\frac{3}{8}\right)^{th}$ rule.

ii. Write the relation between shift operator and forward difference operator.

B) Attempt any two of the following (6)

i. Find $f(2)$ by Lagrange's formula

x	0	1	3	4
$f(x)$	5	6	50	105

ii. Evaluate $\int_0^{1.5} \frac{x}{e^x - 1} dx$, by using trapezoidal rule by dividing [0 , 1.5] into 6 subintervals.

iii. Using Newton backward interpolation formula find the value of $e^{-1.9}$.

C) Attempt any one (4)

i. Find $f'(1.1)$ and $f''(1.1)$ from the following data,

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	0.544	1.296	2.432	4.000

ii. The population of a town in decimal census was given below. Estimate the rate of growth of population for the year 1895.

Year (x)	1891	1901	1911	1921	1931
Population (y)	46	66	81	93	101

Q 4: A) Attempt the following (2)

i. Define probability distribution.

ii. Define expectation of random variable.

B) Attempt any two of the following (6)

i. The probability of an individual coalminer being killed in a mine accident during a year is $1/2400$. Use Poisson distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.

ii. The probability function of random variable x is given in the table. Find the distribution function.

X	1	2	3
$f(x)$	½	1/3	1/6

iii. The distribution function of a random variable X is defined by,

$$F(x) = \begin{cases} cx^2, & 0 \le x \le 3 \\ 1, & x \ge 3 \\ 0, & x < 0 \end{cases}$$

Find c, *density function* $f(x)$ & $P(x > 1)$.

C) Attempt any one (4)

i. If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs. (i) Exactly 2 (ii) More than 5 (iii) at the most 2 (iv) At least 2 bulbs will be defective.

ii. In a lottery there are 100 prizes of Rs. 25/-, 50 prizes Rs. 50/-, 20 prizes of Rs. 100/-, 10 prizes of Rs. 200/- , 4 prizes of Rs. 500/- and one prize of Rs. 1000/-. Assume that 10,000 tickets are to be sold what is the fair prize to pay for the ticket?

Q 5: Attempt any three (12)

i. Fit a straight line $y = a + bx$ to the following data by the method of least square,

X	0	1	3	6	8
y	1	3	2	5	4

ii. Fit a second degree parabola to the following data taking x as an independent variable.

x	1	2	3	4	5	6
y	2	6	7	8	10	11

iii. For the data given below, find the equation to the best fitting exponential curve of the form $y = a.b^x$.

X	1	2	3	4	5
y	5	8	12	16	4

iv. Find the curve of the best fit of the type $y = a.e^{bx}$ to the following data

X	1	2	3	4	5	6
y	1	3	7	12	18	25